

**GEOMETRY – FIELDS FOREVER**

- 1. C**
- 2. A**
- 3. E**
- 4. C**
- 5. C**
- 6. C**
- 7. A**
- 8. C**
- 9. B**
- 10. A**
- 11. B**
- 12. D**
- 13. B**
- 14. A**
- 15. A**
- 16. E**
- 17. D**
- 18. D**
- 19. B**
- 20. A**
- 21. C**
- 22. B**
- 23. C**
- 24. C**
- 25. D**
- 26. C**
- 27. D**
- 28. C**
- 29. A**
- 30. B**

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1. **C.** They go from Tallahassee to Nebraska, from Nebraska to Idaho, from Idaho to Kansas. After plotting all locations of these 4 places, use Pythagorean theorem to find the distances between each locations. Tallahassee to Nebraska = 2500 miles. Nebraska to Idaho = 1250 miles. Idaho to Kansas =  $250\sqrt{41}$  miles.
2. **A.** Split the triangle down the altitude to create 2 congruent right triangles. The right triangles have a hypotenuse of 7 inches, a base of 2 inches, and an unknown height, which is the altitude of the corn. Using Pythagorean theorem,  $(2)^2 + (h)^2 = (7)^2$ ;  $h = 3\sqrt{5}$  inches.
3. **E.** The largest area with perimeter 7927 is in the shape of a circle, which has an infinite number of sides.
4. **C.** Use linear pair with 139 to get 41 for the bottom right angle of the triangle then  $63 + 41 +$  the top angle must equal 180 so the top angle is 76. Using the parallel lines and corresponding angles the angle which forms a linear pair with  $x$  is 76 therefore  $x = 104$ .
5. **C.** Rene-Robert Cavalier, Sieur de La Salle contains 14 distinct letters. Using the diagonal formula,  $(n)(n-3)/2$ , the 14-gon has 77 diagonals.
6. **C.** The triangle DOA can be separated into 3 triangles: DOW, WAD, and WOA. By using the formula  $A = \frac{1}{2}(bh)$ , DOW has an area of 36, WAD has an area of 72, and WOA has an area of 72.  $36 + 72 + 72 = 180$ .
7. **A.** Using shoelace theorem, we can find that the area enclosed by the three points is 10.5. To make the problem more solvable, we can translate the three points such that the coordinates for the world's largest ball of twine is  $(0, 0)$ , making the other coordinates  $(1, 16)$  and  $(3, 69)$ , respectively.
8. **C.** We can view the problem in a 2D manner. The hemisphere can be seen as a semicircle with a radius of 6. Since Vera is 4 ft away from the center, we can form a right triangle, with Vera's height as the unknown leg. Using Pythagorean theorem,  $4^2 + h^2 = 6^2$ ;  $h = 2\sqrt{5}$  ft. However, the problem is asking for her height in inches, so  $2\sqrt{5} \times 12 = 24\sqrt{5}$  inches.
9. **B.** The volume of the entire silo is  $720 \pi \text{ m}^3$ . The volume of the wheat in the silo after leaking is  $4 \cdot ((6)^2) \pi = 144 \pi \text{ m}^3$ . This means that  $576 \pi \text{ m}^3$  of wheat was drained out of the silo, in a duration of 72 minutes. 72 minutes after 12:00 PM is 1:12 PM.
10. **A.** Draw the probability graph on a xy-plane. The cornfield is 2500 sqft and the contamination is 600 sqft so  $600/2500$  simplifies to  $6/25$ .
11. **B.** Use the formula  $(A \times B)/(A + B)$  given that A and B are the heights of the figures, respectively. Therefore,  $(1400)/(150) = 9.333\dots$ , or  $9 \frac{4}{3}$ .
12. **D.** The barn's trapezoidal faces have an area of  $h(b_1 + b_2) / 2 = 12(20 + 30) / 2 = 300 \text{ ft}^2$ . The barn's roof has an area of  $60 \times 20 = 1200 \text{ ft}^2$ . The base has an area of  $60 \times 30 = 1800 \text{ ft}^2$ . The sides of the barn have dimensions of  $60 \times 13$  (13 is solved by Pythagorean theorem of a 5-12-13 triangle) and an area of 780. The entire surface area is  $300 \times 2 + 1200 + 1800 + 780 \times 2 = 5160 \text{ ft}^2$ .
13. **B.** To find the shortest path from a Ammar to the river to the barn, reflect one point (the barn) across the line such that that path from Ammar to the barn is a straight line that intersects the river. The barn's coordinates become  $(-23, 18)$ . The line that represents the path between the barn  $(-23, 18)$  and Ammar  $(5, -3)$  can be solved to be  $y = -\frac{3}{4}x + \frac{3}{4}$ . The intersection between the line and the river

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- $x = -5$  is at the point  $(-5, 4.5)$ , which represents the point where Zuhair should be located. The ordinate of the point  $(-5, 4.5)$  is 4.5.
14. **A.** Jessica's field has 92 heads of lettuce, which means she has  $92/4 = 23$  rows of lettuce and 22 unoccupied rows. One row with lettuce takes up  $4 \times 1 = 4 \text{ ft}^2$ , while one unoccupied row takes up  $4 \times 1.5 = 6 \text{ ft}^2$ . The total field has an area of  $23 \times 4 + 22 \times 6 = 224 \text{ ft}^2$ .
  15. **A.**  $n = 1$  has 0 vertices shared by 3 hexagons,  $n = 2$  has 6 vertices shared by 3 hexagons,  $n = 3$  has 24 vertices shared by 3 hexagons, and  $n = 4$  has 54 vertices shared by 3 hexagons. We see a pattern that the number of vertices shared by 3 hexagons is  $6(n-1)^2$ . Therefore, when  $n = 10$ , there are  $6(9)^2 = 486$  vertices shared by 3 hexagons.
  16. **E.** These are not congruent because SSA is not a valid concurrency theory, so there is no way to know what the angle "must be".
  17. **D.** This obtuse isosceles triangle can be split down the middle to form two 30-60-90 triangles, which have a ratio of side lengths  $1:\sqrt{3}:2$ . So we know the base is  $2\sqrt{17}\sqrt{3}$ . The hypotenuse of the 30-60-90 is  $2\sqrt{17}$ , so 34. Just add up the sides to get  $34 + 34 + 34\sqrt{3}$
  18. **D.** For regular polygons,  $\text{area} = \text{apothem} \times \text{semi-perimeter}$ .  $912 = 8 \times \text{semi-perimeter}$ . Semi-perimeter = 114, so the perimeter = 228. A dodecagon has 12 sides, so each side is  $228/12 = 19$ . The formula for the area of an equilateral triangle is  $s^2 \times \sqrt{3}/4$ .  $= 361\sqrt{3}/4$
  19. **B.** If statement  $p \rightarrow q$  is true, then  $\sim p \rightarrow \sim q$  is also true, by the contrapositive property. The others are false because they could occur given specific circumstances.
  20. **A.** The maximum length is the two sides added to each other, 7, resulting in a degenerate triangle. The minimum is the longer side minus the shorter, leaving  $17/3$ .  $17/3 - 7 = -4/3$ .
  21. **C.** Heron's formula:  $\sqrt{s(s-a)(s-b)(s-c)} = \text{area}$ , where  $s$  is semi-perimeter.  $\sqrt{24(24-14)(24-16)(24-18)} = 48\sqrt{5}$
  22. **B** Medians of a triangle intersect such that the triangle is divided equally into 6 smaller triangles.  $36/6 = 6$ .  $ABE = AGF = 6 \times 3$ .  $BCE = 6$ .  $18+6-18+2020 = 2026$
  23. **C.** Since the length of the base edge is 4, and the length of the top frustum edge is 2, you can use similar triangles to see that the cut was half way up the pyramid (small pyramid has half height of original). This gives you the height of the small pyramid to be 12.  $4 \times 12/3 = 16 \text{ m}^3$  for the small volume.  $16 \times 24/3 = 128 \text{ m}^3$  for the big volume. The frustum is the big pyramid – small pyramid =  $128-16 = 112$
  24. **C.** Area of a trapezoid =  $\text{height} \times (\text{base}_1 + \text{base}_2)/2 \rightarrow (4 + 2)/2 \times 12 = 36 \text{ in}^2$
  25. **D.** Power of a Point.  $BD \times ED = AB^2 \rightarrow 4 \times 16 = x^2 \rightarrow x = 8$ .
  26. **C.** The exterior angles of a regular polygon always add to 360.  $360/24 = 15$  degrees for each exterior angle. The exterior and interior are supplements, so the interior angle is 165. There are 24 interior angles so:  $165 \times 24 = 3960$
  27. **D.** Every corner of the barn is a new center for part of circle of available space. You get one three quarter circle, and 3 quarter circles with radii 8, 5, 4, and 1.  $\pi(3/4 \times 64 + 1/4 \times 25 + 1/4 \times 4 + 1/4) = 55.5 \pi$
  28. **C.** Cone volume =  $1/3 \times \text{base} \times \text{height}$ .  $\text{Height} = 3 \times \text{Volume} / \text{Base} \rightarrow 3 \times 216000 / 120 = 5400$

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29. **A.** Maximum number of intersections =  $n(n-1)/2$ . This can be found by finding a pattern.  $12 \cdot 11/2 = 66$ . Minimum = 1 point because they are not parallel.  $66-1 = 65$
30. **B.** There were three people, so they each had  $1/3$  pie. When Brandon showed up, we have four people, so we need everyone to have  $1/4$  pie. If the three original people each donate  $1/12$  of the original pie from their share to Brandon, all four friends will have  $3/12$  of the pie. The arc length is just a fraction of the circumference. Since they donated  $1/12$  the pie, they will have  $2\pi r/12 = \pi r/6$